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Solution of Duffin–Kemmer–Petiau Equation for the Step Potential

L. Chetouani,[1] M. Merad,[2] T. Boudjedaa,[3] and A. Lecheheb[1]

The Duffin–Kemmer–Petiau (DKP) equation for spin 0 and 1 in the pressence of the step potential is solved. The problem is reduced to the solution of an equation of Feshbach–Villars type.The reflection and transmission coefficients are correctly deduced. The Klein paradox persists and is discussed using the charge interpretation.

KEY WORDS: Duffin–Kemmer–Petiau equation; step potential.

1. INTRODUCTION

The Duffin–Kemmer–Patiau (DKP) equation is a natural way to describe scalar and vector particles with the help of a covariant relativistic formalism (Petiau, 1936; Duffin, 1938; Kummer, 1939). It counts among the many attempts which followed the exploit of the Dirac theory of the particle of spin $\frac{1}{2}$ with an aim of describing particles of spin 0 and 1. In other wards, the DKP equation is a direct generalization of the equatioin of Dirac to the particles of integer spin in which one replaces the γ matrices by β matrices but verifying a more complicated algebra known as DKP algebra. Although the DKP equation contains in it the description of the scalar particles of spin 0, it is not completely equivalent to the equation of Klein–Gordon (KG) except if the interaction is absent. This is due to the following fact: when one couples the particle with the field of interaction by means of the minimum coupling and one squares this equation, then appears an anomaly term entirely without physical meaning and which, moreover, breaks apparently the gauge invariance. This major defect played against the DKP equation, excluding it for a long time from competition. For the past few years there was a renewed interest toward this equation. It has been applied to the quark confinement problem of quantum chromodynamics theory (Gribov, 1999). Also let us note that several efforts were provided with an aim of restoring this equivalence (Fainberg and

[1] Départément de Physique, Faculté des Sciences, Université Mentouri, 25000 Constantine, Algéria.
[2] Départément de Physique, Centre Universitaire de Oum-El-Bouaghi, 04000 Oum-El-Bouaghi, Algeria.
[3] Départément de Physique, Centre Universitaire de Jijel, BP 98, Ouled Aissa, 18000 Jijel, Algeria.

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Pimentel, 2000) by showing that this contradiction is only apparent (Luanrdi *et al.*, 2000) and can be elucidated by a correct interpretation of the DKP theory. This equation has also been generalized to the case of curved spaces (Lunardi, Pimentel, and Teixeiri, 1999). Without forgetting some works in which one considered the exact solutions of this equation in the presence of an external field (Nedjadi and Barrett, 1993). It is clear that these solutions are of a paramount interest in the study of the physical processes. In addition, the DKP equatioin offers a revival in the hope to find a positive density of probability for the particles of spin 0 following the example that of Feshbach–Villars (FV) (Feschbach and Villars, 1958).

The aim of this paper is to find the exact solutions of the DKP equation in the presence of the step potential. As it is easy to see it, a naive treatment of the boundary conditions brings us directly to the trivial solution. This fact then obliges us to replace the step potential by a smooth potential. The passage to the limit enables us to find the good boundary conditiions and thus to extract the exact solutions. This technique was already used for the FV equation in the treatment of the potential of this kind (Merad, Chetouani, and Bounames, 2000; Bounames and Chetouani, 2001). This is not a fortuitous fact because as one will see it when one reduces the DKP equation to that of KG, a system of equation identical to that of FV appears then. In Section 2 we are interested by the solution of the equation in the case of spin 1. It is shown initially that it is possible to reduce the DKP equation to that of KG. Consequently, the solution in the case of the smooth potential is obtained and the results of the step potential are deduced by the passage to the limit. Following the same method, one determines the exact solutions of the DKP equation in the case of spin 0 as a particular case (Section 3). It is remarkable that in both cases, the coefficients of reflection and transmission are identical.

Before starting the resolution of the DKP equation, we will initially point out some formulas useful for the later developements. The DKP equation describing the particles of spin 0 and 1 interacting with an electromagnetic field is similar to that of Dirac, namely

$$[i\beta^{\mu}(\partial_{\mu} + ieA_{\mu} - m]\psi(x, t) = 0 \tag{1}$$

where the matrices β^{μ} are singular and verify the following commutation relations

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\nu\lambda}\beta^{\mu} \tag{2}$$

From these equations, it is easy to define the adjoint spinor $\overline{\Psi}$ by

$$\overline{\psi} = \psi^{+}\left(2\beta_0^2 - \mathbf{1}\right) \tag{3}$$

which verifies the following adjoint equation

$$i(\partial_{\mu} - ieA_{\mu})\overline{\psi}\beta^{\mu} + m\overline{\psi} = 0 \tag{4}$$

In consequence, from the Eqs. (1) and (4), we can obtain the continuity equation as

$$\partial_\mu J^\mu = 0 \tag{5}$$

where $J^\mu \equiv \bar{\psi}\beta^\mu\psi$.

Let us notice that the density J^0 is not positively defined and we can follow the reinterpretation besed on the charge symmetry initiated by Pauli and Weisskopf and taken again by Feshbach and Villars. Then to be able to explain the Klein paradox, it is necessary to multiply J^0 by the elementary charge e.

The matrices β^μ generate an algebra known as DKP algebra which in principle have three inrreducible representations. The representation of dimension 1, known as trivial one, does not correspond to any type of particles. On the other hand, those of demensoin 5 and 10 describe the particles of spin 0 and 1 respectively.

1. For the case of spin 0, the explicit form of β^μ is given by

$$\beta^0 = \begin{pmatrix} \theta & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad \beta^i = \begin{pmatrix} 0 & \rho^i \\ -\rho_T^i & 0 \end{pmatrix}, \quad i = 1, 2, 3 \tag{6}$$

where the block elements are defined as

$$\theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{7}$$

and

$$\rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

with ρ^{T} denotes the transposed matrix of ρ and $\mathbf{0}$ is the zero matrix.

2. For the case of spin 1, the explicit form of β^μ is given as

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^{\mathrm{T}} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^{\mathrm{T}} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad \text{and} \quad \beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^{\mathrm{T}} & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix} \quad i = 1, 2, 3 \tag{9}$$

where the matrices s_i are the standard nonrelativistic (3×3) spin 1 matrices and, $\mathbf{0}$ and $\mathbf{1}$ denote the zero matrix and the unit matrix respectively. The matrices $\bar{0}$ and e_i are given as

$$\bar{0} = (0\ 0\ 0), \quad e_1 = (1\ 0\ 0), \quad e_2 = (0\ 1\ 0), \quad \text{and } e_3 = (0\ 0\ 1) \tag{10}$$

In what follows we are interested by the resolution of the DKP equation respectively for the particles of spin 0 and 1, in the case of the intereaction with a scalar step potential $V(z) = V_0\theta(z)$. Let us notice that for this potential the DKP equation gives with the habitual boundary conditons, the continuity on the wave function and its derivative $\psi(0^+) = \psi(0^-)$ and $\psi'(0^+) = \psi'(0^-)$, gives directly to the trivial solution $\psi = 0$.

In order to circumvent this problem, it is convenient to replace the step potential by a smooth potential. In our case we choose for this potential the following form

$$V(z) = \frac{V_0}{2}\left(1 + \tanh\frac{z}{2r}\right) \tag{11}$$

where V_0 and r are positive parameters. The step potential is obtained by taking the limit $r \longrightarrow 0$, namely, $V(z)_{\lim\ \mathrm{r}\longrightarrow 0} \longrightarrow V_0\theta(z)$. In this case, the DKP equation is reduced to

$$\left[i\beta^0\left(\frac{\partial}{\partial t} + ieV\right) + i\beta^3\frac{\partial}{\partial z} - m\right]\psi(z,t) = 0 \tag{12}$$

As the potential is independent from the time t, we have then to find the satationary states of this equation. Accordingly, let us choose for $\psi(z,t)$ the following form $e^{-iEt}\phi(z)$ and then get the following eigenvalue equation

$$\left[\beta^0(E - eV) + i\beta^3\frac{d}{dz} - m\right]\phi(z) = 0 \tag{13}$$

2. SOLUTION OF THE DKP EQUATION FOR SPIN 1

It is obvious that the DKP equation, as a relativistic equation, is fundamentally related to that of KG. Indeed, as we can see it, the equations of the system (13) are not completely independent. The wave function $\phi(z)^{\mathrm{T}}$has ten components (φ, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) with $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are vectors of dimension (3×1) which can be decomposed $\phi(z)^{\mathrm{T}}$ as follows

$$\Psi^{\mathrm{T}} = (A_1, A_2, B_3), \quad \Phi^{\mathrm{T}} = (B_1, B_2, A_3), \quad \Theta^{\mathrm{T}} = (C_2, -C_1, \varphi) \text{ and } C_3 \tag{14}$$

where A_i, B_i, $and\ C_i\ i = 1, 2, 3$ are respectively the components of the vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

With these notations, it is not difficult to verify that only $\mathbf{\Psi}$ components are independents and satisfy the following KG type equation

$$\mathbf{O}_{\mathrm{KG}}\Psi = 0 \tag{15}$$

The scalar differential operator $\mathbf{O}_{\mathrm{KG}}$ is the corresponding KG operator defined as

$$\mathbf{O}_{\mathrm{KG}} = \frac{d^2}{dz^2} + [(E - eV)^2 - m^2] \tag{16}$$

The other components are determined by the following constraints equations

$$\begin{pmatrix} \Phi \\ \Theta \end{pmatrix} = \begin{pmatrix} \dfrac{(E - eV)}{m} \\ \dfrac{i}{m}\dfrac{d}{dz} \end{pmatrix} \otimes \boldsymbol{\Psi} \tag{17}$$

The component C_3 automatically vanishes ($C_3 = 0$).

It is remarkable to note that the subcomponents $(\boldsymbol{\Psi}, \boldsymbol{\Phi})^{\mathrm{T}}$ of $\phi(z)^{\mathrm{T}}$ play a similar role to that of the sum and difference of two components wave function of FV equation (Merad, Chetouani, and Bounames, 2000; Bounames and Chetouani, 2001). So, we could claim that these two components express the charge symmetry existing in the problem.

Now, in order to solve the Eq. (15), let us introduce the change variable

$$y = \frac{1}{2}\left(1 - \tanh\frac{z}{2r}\right) \tag{18}$$

where y vary in the domain [0, 1]. The new form of the Eq. (15) will be written as

$$\frac{1}{r^2}y^2(1-y)^2\frac{d^2\Psi}{dy^2} + \frac{1}{r^2}y(1-y)(1-2y)\frac{d\Psi}{dy} + [(E - eV_0(1-y))^2 - m^2]\Psi = 0 \tag{19}$$

In addiltion, we note that this equation possesses three singular points $y = 0, 1, \infty$. By means of the substitution $\Psi = y^{\nu}(1-y)^{\mu}\tilde{\Psi}$, this equation will reduce to the hypergeometric type

$$y(1-y)\frac{d^2\tilde{\Psi}}{dy^2} + [(2\nu + 1) - y(2\nu + 2\mu + 2)]\frac{d\tilde{\Psi}}{dy} + \left[\left(\mu + \nu + \frac{1}{2}\right)^2 - \frac{\nu_0^2}{4}\right]\tilde{\Psi} = 0 \tag{20}$$

where $\nu^2 = r^2[m^2 - (E - eV_0)^2]$, $\mu^2 = r^2[m^2 - E^2]$, and

$$\nu_0 = \sqrt{(1 - 2reV_0)(1 + 2reV_0)}.$$

The regular sulution at origin $y = 0$ of this differential equation is given in terms of hypergeometric functions as

$$\Psi(y) = y^{\nu}(1-y)^{\mu}\,{}_2F_1\left(\mu + \nu + \frac{1}{2} - \frac{\nu_0}{2}, \mu + \nu + \frac{1}{2} + \frac{\nu_0}{2}, 1 + 2\nu, y\right)\mathbf{V} \tag{21}$$

where $\mathbf{V}$ is a constant vector of dimension (3×1).

Taking into account of the following hypergeometric property

$$\frac{d_2F_1(\alpha,\beta,\gamma,y)}{dy}=\frac{\alpha\beta}{\gamma}\ {}_2F_1(\alpha+1,\beta+1,\gamma+1,y) \tag{22}$$

we will obtain the solution written as

$$\begin{pmatrix}\Psi\\ \Phi\\ \Theta\end{pmatrix}=y^{\nu}(1-y)^{\mu}[{}_2F_1(\alpha,\beta,\gamma,y)\mathbf{M}(y)+\ {}_2F_1(\alpha+1,\beta+1,\gamma+1,y)\mathbf{N}(y)] \tag{23}$$

with $\mathbf{M}(y)$ and $\mathbf{N}(y)$ are (9×1) components vectors defined as

$$M(y)=\begin{pmatrix}1\\ \dfrac{E-eV_0(1-y)}{m}\\ \dfrac{-i[\nu-(\mu+\nu)y]}{mr}\end{pmatrix}\otimes\mathbf{V}\quad\text{and}\quad \mathbf{N}(y)=\begin{pmatrix}0\\ 0\\ \dfrac{-i\alpha\beta}{mr\gamma}y(1-y)\end{pmatrix}\otimes\mathbf{V} \tag{24}$$

where $\alpha=\mu+\nu+\frac{1}{2}-\frac{\nu_0}{2}$, $\beta=\mu+\nu+\frac{1}{2}+\frac{\nu_0}{2}$, and $\gamma=1+2\nu$.

In fact and as it easy to see, the components of the vector $\mathbf{V}(i=1,2,3)$ are the constants relative to the three independent directions of spin 1. Now, by returning to the old variable z, we obtain from (23) the final result

$$\begin{aligned}\begin{pmatrix}\Psi\\ \Phi\\ \Theta\end{pmatrix}&=\left[\frac{1}{2}\left(1-\tanh\frac{z}{2r}\right)\right]^{\nu}\left[\frac{1}{2}\left(1+\tanh\frac{z}{2r}\right)\right]^{\mu}\\ &\times\left[{}_2F_1\left(\alpha,\beta,\gamma,\frac{1}{2}\left(1-\tanh\frac{z}{2r}\right)\right)\mathbf{M}(z)\right.\\ &\left.+\,{}_2F_1\left(\alpha+1,\beta+1,\gamma+1,\frac{1}{2}\left(1-\tanh\frac{z}{2r}\right)\right)\mathbf{N}(z)\right]\end{aligned} \tag{25}$$

with

$$M(z)=\begin{pmatrix}1\\ \dfrac{E-\frac{eV_o}{2}\left(1+\tanh\frac{z}{2r}\right)}{m}\\ \dfrac{-i\left[\nu-\frac{(\mu+\nu)}{2}\left(1-\tanh\frac{z}{2r}\right)\right]}{mr}\end{pmatrix}\otimes\mathbf{V} \tag{26}$$

$$N(z) = \begin{pmatrix} 0 \\ 0 \\ \dfrac{-i\alpha\beta}{4mr\gamma}\left(1 - \tanh\dfrac{z}{2r}\right)\left(1 + \tanh\dfrac{z}{2r}\right) \end{pmatrix} \otimes \mathbf{V} \qquad (27)$$

With the aim of extracting the reflection and transmission coefficients relating to the smooth potential and in the same way the wave function of step potential, let us study the asymptotic behavior of the wave function at $z \longrightarrow \pm\infty$.

For $z \longrightarrow +\infty$ (or $y \longrightarrow 0$), the wave function will have the following behavior

$$\begin{pmatrix} \Psi \\ \Phi \\ \Theta \end{pmatrix}_{z\longrightarrow+\infty} = e^{-\nu z/r}\left[\begin{pmatrix} 1 \\ \dfrac{E - eV_0}{m} \\ \dfrac{-i\nu}{mr} \end{pmatrix} \otimes \mathbf{V}\right] \qquad (28)$$

where we have used the limits

$$\lim_{y\longrightarrow 0} y^{\nu} = e^{-\nu z/r}, \quad \lim_{y\longrightarrow 0}(1 - y)^{\mu} = 1 \quad \text{and} \quad \lim_{y\longrightarrow 0}\, {}_2F_1(\alpha, \beta, \gamma, y) = 1 \qquad (29)$$

For $z \longrightarrow -\infty$(or $y \longrightarrow 1$) , we use the hypergeometric property

$$\begin{aligned} {}_2F_1(\alpha, \beta, \gamma, y) &= A\, {}_2F_1(\alpha, \beta, \alpha + \beta - \gamma + 1, 1 - y) \\ &\times B(1 - y)^{\gamma-\alpha-\beta}\, {}_2F_1(\gamma - \alpha, \gamma - \beta, \gamma - \alpha - \beta + 1, 1 - y) \end{aligned} \qquad (30)$$

where

$$A = \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} \quad \text{and} \quad B = \frac{\Gamma(\gamma)\Gamma(\alpha + \beta - \gamma)}{\Gamma(\alpha)\Gamma(\beta)} \qquad (31)$$

to get

$$\begin{pmatrix} \Psi \\ \Phi \\ \Theta \end{pmatrix}_{z\longrightarrow-\infty} = \left[\begin{pmatrix} Ae^{\mu z/r} + Be^{-\mu z/r} \\ \dfrac{E}{m}(Ae^{\mu z/r} + Be^{-\mu z/r}) \\ \dfrac{i\mu}{mr}(Ae^{\mu z/r} - Be^{-\mu z/r} \end{pmatrix} \otimes \mathbf{V}\right] \qquad (32)$$

where we have used the limits

$$\begin{aligned} &\lim_{y\longrightarrow 1} y^{\nu} = 1, \quad \lim_{y\longrightarrow 1}(1 - y)^{\mu} = e^{\mu z/r}, \quad \lim_{y\longrightarrow 1}(1 - y)^{-\mu} \quad e^{-\mu z/r}, \\ &= \lim_{y\longrightarrow 1}\, {}_2F_1(\alpha, \beta, \gamma, 1 - y) = 1 \end{aligned} \qquad (33)$$

At this stage, it is easy to calculate the reflection and transmission coefficients along the direction of the spin $\mathbf{V}_i$, related to the smooth potential. In effect, by using the definition of the quadrivector density current of probabilitiy, we obtain

$$R = \frac{|J_{\text{ref}}|}{|J_{\text{inc}}|} = \left| \frac{\Gamma\left(\nu + \mu + \frac{v_0}{2} + \frac{1}{2}\right)\Gamma\left(\nu + \mu - \frac{v_0}{2} + \frac{1}{2}\right)}{\Gamma\left(\nu - \mu + \frac{v_0}{2} + \frac{1}{2}\right)\Gamma\left(\nu - \mu - \frac{v_0}{2} + \frac{1}{2}\right)} \right|^2 \tag{34}$$

It is noted that the term $|\frac{\Gamma(-2\mu)}{\Gamma(2\mu)}|$ does not contribute to R because the two terms are complex conjugates. Then, the expression of T has the following form

$$T = \frac{|J_{\text{tr}}|}{|J_{\text{inc}}|} = \frac{|\nu - \nu^*|\left|\exp\left[-\frac{z}{r}(\nu + \nu^*)\right]\right|}{2|\mu|} \left| \frac{\Gamma\left(\nu + \mu + \frac{v_0}{2} + \frac{1}{2}\right)\Gamma\left(\nu + \mu - \frac{v_0}{2} + \frac{1}{2}\right)}{\Gamma(2\mu)\Gamma(1 + 2\nu)} \right|^2 \tag{35}$$

Now, let us turn to the step limit. Taking the limit $r \longrightarrow 0$, we readily obtain from (25) the wave function of the step potential

$$\begin{pmatrix} \Psi \\ \Phi \\ \Theta \end{pmatrix} = [\theta(z)\mathbb{V}^{\text{tr}} e^{ik_2 z} + \theta(-z)(\mathbb{V}^{\text{inc}} e^{ik_1 z} + \mathbb{V}^{\text{ref}} e^{-ik_1 z}] \otimes \mathbf{V} \tag{36}$$

with the vectors $\mathbb{V}^{\text{tr}}$, $\mathbb{V}^{\text{inc}}$, and $\mathbb{V}^{\text{ref}}$ are defined as

$$\mathbb{V}^{\text{inc}} = \frac{k_1 + k_2}{2k_1} \begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{-k_1}{m} \end{pmatrix}, \quad \mathbb{V}^{\text{ref}} = \frac{k_1 - k_2}{2k_1} \begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{k_1}{m} \end{pmatrix}, \quad \mathbb{V}^{\text{tr}} = \begin{pmatrix} 1 \\ \frac{E - eV_0}{m} \\ \frac{-k_2}{m} \end{pmatrix} \tag{37}$$

To get previous expressions we have used

$$\lim_{r \longrightarrow 0} A = \frac{k_1 - k_2}{2k_1} \quad \text{and} \quad \lim_{r \longrightarrow 0} B = \frac{k_1 + k_2}{2k_1} \tag{38}$$

with $\nu = -irk_2$, and $k_2^2 = [(E - eV_0)^2 - m^2]$, and $\mu = -irk_1$ with $k_1^2 = E^2 - m^2$ and k_1 is real. We notice that k_2 is real for $E < eV_0 - m$ *or* $E > eV_0 + m$ and is imaginary for $eV_0 - m < E < eV_0 + m$. It is remarkable to note that $\mathbb{V}^{\text{ref}}$and $\mathbb{V}^{\text{tr}}$ are respectively obtained from $\mathbb{V}^{\text{inc}}$ by changing $k_1 \longrightarrow -k_1$and $k_1 \longrightarrow k_2$.

The reflection and transmission coefficients of the step potential can be now easilly deduced. The result, in different energy region, is

– for $eV_0 - m < E < eV_0 + m$, (k_2 is imaginary)

$$R = 1 \quad \text{and} \quad T = 0 \tag{39}$$

– for $E > eV_0 + m$, (k_2 is real)

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}, \quad T = \frac{4k_1 k_2}{(k_1 + k_2)^2}, \quad \text{and } R + T = 1 \tag{40}$$

– for $E < eV_0 - m$, (k_2 is real)

$$R = \frac{(k_1 + k_2)^2}{(k_1 - k_2)^2}, \quad T = \frac{4k_1 k_2}{(k_1 - k_2)^2}, \quad \text{and } R + T = 1 \tag{41}$$

We note that in this case we encounter the famous Klein paradox as in the KG and FV theories (Boudjedda, Chetouani, and Merad, 1999).

3. BOUNDARY CONDITIONS

In this section, we are going to determine the good boundary conditions for the potential admitting a jump at unspecified point z_0 Let $V(z)$ be such a potential

$$V(z) = \begin{cases} V_1(z) & \text{for } z < z_0 \\ V_2(z) & \text{for } z < z_0 \end{cases} \tag{42}$$

As it is has been said previously the naive conditions of continuity lead directly to the trivial solution. To find the adequate conditions, we proceed in the following way. Let us start from Ψ which satisfied the KG equation (15). Then we must impose on it and on its derivative the continuity conditions. By integrating Eq. (15) in the domain $[z_0^-, z_0^+]$, one gets

$$\Psi(z_0^+) = \Psi(z_0^-) \tag{43}$$

$$\frac{d\Psi(z_0^+)}{dz} = \frac{d\Psi(z_0^-)}{dz} \tag{44}$$

Using these conditions (43) and (44) we obtain

$$\begin{pmatrix} \Psi(z_0^+) \\ \Phi(z_0^+) \\ \Theta(z_0^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{E - eV_2(z_0^+)}{E - eV_1(z_0^-)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Psi(z_0^-) \\ \Phi(z_0^-) \\ \Theta(z_0^-) \end{pmatrix} \tag{45}$$

These conditions are determined by taking only one direction of propagation of the spin along one axis. It is easy to verify that the solution given by the smooth potential checks these conditions when $r \longrightarrow 0$. This is identical to the case of photon (DKP massless particle) traversing two different regions. As it is known, the tangential electric component is continuous end the normal magnetic component is discontinuous.

In addition, these boundary conditions are reflected on the charge density and the current density of charge like

$$J^0(z_0^+) = \frac{E - eV_2(z_0^+)}{E - eV_1(z_0^-)} J^0(z_0^-) \tag{46}$$

$$J^i(z_0^+) = J^i(z_0^-) \tag{47}$$

i.e., that the current density of charge along i-axis remains always continuous whereas the charge density has a discountinuity at the point of the jump z_0 of the potential. Eq. (47) ensures the conservation of the total charge, whereas the multiplicative factor present in Eq. (46) enables us then according to its sign to determine if there is creation of pair particle–antiparticle or not. This equation reveals then the presence of the pair creation and permits to elucidate the Klein paradox. In Eq. (47) there appears, in principle, an inversion of the sign indicating then the manifestation of the antiparticle on the other side of jump of the barrier potential. As the DKP equation concerns bosons, it is not then necessary to introduce the Dirace sea to explain the Klein paradox, and we will recourse consequently to the charged DKP field to explain this effect.

4. SOLUTION OF THE DKP EQUATION FOR SPIN 0

As we will see, the study of the case of spin 0 is similar to that of spin 1. In effect, by putting $\phi(z)^{\mathrm{T}} = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$, the system equations (13) is brought back to the following system

$$\begin{cases} \mathbf{O}_{\mathrm{KG}}\eta_1 = 0 \\ \eta_2 = \dfrac{(E - eV)}{m}\eta_1 \\ \eta_3 = 0 \\ \eta_4 = 0 \\ \eta_5 = \dfrac{i}{m}\dfrac{d\eta_1}{dz} \end{cases} \tag{48}$$

which indicates the following correspondence: $\eta_1 \longrightarrow \Psi$, $\eta_2 \longrightarrow \Phi$, $\eta_5 \longrightarrow \Theta$ and $(\eta_3, \eta_4) \longrightarrow C_3$. Then, we will write the solution of (48) as

$$\begin{pmatrix} \eta_2 \\ \eta_5 \end{pmatrix} = \begin{pmatrix} \dfrac{(E - eV)}{m} \\ \dfrac{i}{m}\dfrac{d}{dz} \end{pmatrix} \otimes \eta_1 \tag{49}$$

with

$$\eta_1(y) = Cy^{\nu}(1-y)^{\mu}\ {}_2F_1\left(\mu+\nu+\frac{1}{2}-\frac{v_0}{2}, \mu+\nu+\frac{1}{2}+\frac{v_0}{2}, 1+2_{\nu}, y\right) \tag{50}$$

C is a constant. The global solution is

$$\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_5\end{pmatrix} = \left[\frac{1}{2}\left(1-\tanh\frac{z}{2r}\right)\right]^{\nu}\left[\frac{1}{2}\left(1+\tanh\frac{z}{2r}\right)\right]^{\mu} \times\left[{}_2F_1\left(\alpha,\beta,\gamma,\frac{1}{2}\left(1-\tanh\frac{z}{2r}\right)\right)\mathbf{K}(z) + {}_2F_1\left(\alpha+1,\beta+1,\gamma+1,\frac{1}{2}(1-\tanh\frac{z}{2r}\right)\right)\mathbf{L}(z)\right] \tag{51}$$

with

$$K(z) = C\begin{pmatrix} 1 \\ \dfrac{E-\frac{eV_0}{2}\left(1+\tanh\frac{z}{2r}\right)}{m} \\ \dfrac{-i\left[\nu-\frac{(\mu+\nu)}{2}\left(1-\tanh\frac{z}{2r}\right)\right]}{mr}\end{pmatrix} \tag{52}$$

$$N(z) = C\begin{pmatrix} 0 \\ 0 \\ \dfrac{-i\alpha\beta}{4mr\gamma}\left(1-\tanh\dfrac{z}{2r}\right)\left(1+\tanh\dfrac{z}{2r}\right)\end{pmatrix} \tag{53}$$

A similar calculation to that of spin 1 gives respectively for the coefficients R and T, the same expressions (34) and (35) and the wave function of the step (the limit $r \longrightarrow 0$)

$$\begin{pmatrix}\eta_1\\ \eta_2\\ \eta_5\end{pmatrix} = C[\theta(z)\mathbb{U}^{\text{tr}}e^{ik_2z} + \theta(-z)(\mathbb{U}^{\text{inc}}e^{ik_1z} + \mathbb{U}^{\text{ref}}e^{-ik_1z})] \tag{54}$$

with the vectors $\mathbb{V}^{\mathrm{tr}}$, $\mathbb{V}^{\mathrm{inc}}$, and $\mathbb{V}^{\mathrm{ref}}$ are defined as

$$\mathbb{U}^{\mathrm{inc}} = \frac{k_1+k_2}{2k_1}\begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{-k_1}{m} \end{pmatrix}, \quad \mathbb{U}^{\mathrm{ref}} = \frac{k_1-k_2}{2k_1}\begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{k_1}{m} \end{pmatrix}, \quad \mathbb{U}^{\mathrm{tr}} = \begin{pmatrix} 1 \\ \frac{E-eV_0}{m} \\ \frac{-k_2}{m} \end{pmatrix} \tag{55}$$

The suitable boundary conditions are also deduced in this case as,

$$\begin{cases} \eta_1(0^+) = \eta_1(0^-) \\ \eta_2(0^+) = \dfrac{E-eV_0}{E}\eta_2(0^-) \\ \eta_5(0^+) = \eta_5(0^-) \end{cases} \tag{56}$$

In this case, the Klein paradox also persists and is solved using the same argument. Let us finally notice that for photon there is no paradox.

5. CONCLUSION

We have solved the DKP equation in the case of the smooth potential. The good boundary conditions were found and tested in the case of the step potential. The DKP equation was reduced to that of KG. The obtained system is similar to that of FV indicating that there is a close link between the two formalisms. The wave functions were given in both cases of spin 1 and 0 and the coefficients of reflection and transmission were correctly determined and are identical. The Klein paradox persists in the DKP equation and there is no necessity to introduce the Dirac sea. The explanation would in principle be based on the quantum DKP field.

Finally, let us notice that in the limit of the null masse for the case of spin 1, the DKP equation reduces to an equation with six components (Maxwell equations), the potential playing the role of a source. A analogy of these coefficients for the electromagnetic waves which cross the surface separating the two mediums of indices n_1 and n_2 can be establilshed Jackson (1975).

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